

The Standard Error of the Mean at Different Levels of Aggregation

The mean rating can be reported for each class, for each department, for each college, and for the whole university. However, the range of uncertainty in the reported means at each level of aggregation will vary as a function of the size of the *standard error of the mean* at that level. The generic formula for computing the standard error at each level is shown below. It is computed by taking the square root of the relative error variance for the level at which the reported means have been aggregated. The specific elements included in the formula very depending upon the level of aggregation. However, the basic form is the same at all levels. This form consists of two basic parts:

1. Measurement Error Variance
2. Sampling Error Variance

$$\text{Standard Error of the Mean} = \sqrt{(\text{Sum of the Applicable Measurement Error Variance Components}) + (\text{Sum of the Applicable Sampling Error Variance Components})}$$

Conceptually, the standard error of the mean at each level describes the variability (or heterogeneity) among the sub-unit means about the unit mean at that level of aggregation. As shown in the example below, the formula for the standard at level 6 includes three measurement error variance components added together plus two sampling error variance components. At each higher level, one new measurement error variance component is added to the formula, and one more sampling error variance component is included. Each variance component is a fraction which includes a model-estimated variance parameter in the numerator and one or more integer values in the denominator. At the lowest level of aggregation in each column there is only one term in the denominator, but one additional term is added to the denominators in that column at each higher level in successive steps.

Measurement Error Variance Components

		Items (σ_e^2)	Students (σ_d^2)	Classes (σ_f^2)	Departments (σ_g^2)	Colleges (σ_h^2)
Levels of Aggregation	Level 8 (University)	$\frac{\sigma_e^2}{r * p * q * n * m}$	$\frac{\sigma_d^2}{r * p * q * m}$	$\frac{\sigma_f^2}{r * p * m}$	$\frac{\sigma_g^2}{r * m}$	$\frac{\sigma_h^2}{m}$
	Level 7 (Colleges)	$\frac{\sigma_e^2}{p * q * n * m}$	$\frac{\sigma_d^2}{p * q * m}$	$\frac{\sigma_f^2}{p * m}$	$\frac{\sigma_g^2}{m}$	
	Level 6 (Depts.)	$\frac{\sigma_e^2}{q * n * m}$	$\frac{\sigma_d^2}{q * m}$	$\frac{\sigma_f^2}{m}$		
	Level 4 (Classes)	$\frac{\sigma_e^2}{n * m}$	$\frac{\sigma_d^2}{m}$			
	Level 2 (Students)	$\frac{\sigma_e^2}{m}$				
	Level 1 (Items)					

Sampling Error Variance Components

Students (τ_π)	Classes (τ_β)	Departments (τ_γ)	Colleges (τ_δ)
$\frac{\tau_\pi}{r * p * q * n}$	$\frac{\tau_\beta}{r * p * q}$	$\frac{\tau_\gamma}{r * p}$	$\frac{\tau_\delta}{r}$
$\frac{\tau_\pi}{p * q * n}$	$\frac{\tau_\pi}{p * q}$	$\frac{\tau_\pi}{p}$	
$\frac{\tau_\pi}{q * n}$	$\frac{\tau_\pi}{q}$		
$\frac{\tau_\pi}{n}$			

Example: Standard Error at the Department Level

$$SE_{\bar{X}_{dept}} = \sqrt{\left(\frac{\sigma_e^2}{q * n * m} + \frac{\sigma_d^2}{q * m} + \frac{\sigma_f^2}{m}\right) + \left(\frac{\tau_\pi}{q * n} + \frac{\tau_\beta}{q}\right)}$$

Where:

m = number of questions in the rating form
n = number of responding students in a class
q = number of classes taught within a dept.
p = number of departments within a college
r = number of colleges within the university